ANALYSIS OF TWO STAGE SUSPENSION RAILWAY VEHICLES BASED ON OMA METHOD

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1. Background

EAFIT University, through the Research Group GEMI, has decided to continue a series of Investigation Projects aimed at innovation, science and technology in the field of railway systems. The knowhow on trains constitutes the basis on which EAFIT University develops Investigation Project directed to companies in the transport sector and co-financed by the Administrative Department of Science, Technology and Innovation of Colombia.

It is in this direction that the work is focused on a methodological approach for the evaluation of the railway vehicles parameters, performed through a modal analysis under commercial conditions of the technical system. A methodology for the modal analysis of railway systems is formulated and applied. In order to comply with the objectives proposed for the project it is necessary to perform different activities which can be synthesized in the following fronts: (i) definition of the characteristics of the railway systems as the subject of study, (ii) determination of the security and comfort criteria for the evaluation of the railway system through the modal analysis, (iii) construction of the railway systems numerical model, and (iv) development of the case study.

2. Introduction

The modal properties of the railway systems are traditionally obtained using techniques known as EMA. EMA techniques have been widely documented **[1–3]**, nonetheless it requires the system to be artificially excited and rarely does that excitation have the characteristics that exist in the railway system under commercial condition. As a matter of fact, the properties estimated by EMA may be poor relative to the response in service **[4, 5]**.

The OMA technique is based on the measurement of the response signals only, using environmental excitations as unmeasured signals and obtaining the modal identification with high precision under typical operating conditions **[6, 7]**. The advantages it possesses are the following: (i) evaluate highly non-linear systems (*creepage* phenomenon) for which the only parameter that can measured exactly are the response data **[8, 9]**; (ii) realize *in situ* tests without interruption and in parallel with other applications, it avoids the necessity of having an express test bank for the systems or its component and the dynamic system to be subject inactive time **[10]**; and (iii) it is a simple test procedure similar to the Operating Deflection Shapes (ODS) technique, which employs a reference transducers and various mobile response transducers. OMA does not use the reference transducers in the case that all the response are measured simultaneously, on the contrary, it uses one or various reference transducers depending of the number of repeated roots obtained. Any measurement can be employed as reference, meaning that OMA is a Multi-Input Multi-Output (MIMO) technique and allows estimating very proximal modes with high precision **[8, 10]**.

OMA-LSCE technique has been adopted and in this approach the modal parameters of the system are identified using standard methods of identification in the domain of time known as LSCE **[10, 11]**, which determines the relation between the Impulse Response Function (IRF) in a system of Multiple Degrees of Freedom (MDoF), its complex poles and residues through a complex exponential. IRF can be derived from the inverse of the Fourier Transform (FT) of a Frequency Response Function (FRF), through the process of Random Decrement (RD), or other methods. An Auto-Regressive (AR) model is built using the relationship between poles and residues; the solution of the AR model allows defining a polynomial expression in which the roots of the mathematical system belongs to the complex. Having established the roots (equivalent to the natural frequencies Ω, and the damping rates ξ) the v may be derived by the AR model and obtain the modal shapes φ_i **[2]**.

With the calculation of the poles of the system it is possible to built de stabilization diagram that allows to represent graphically the poles when it is excited in one point (reference) and measurements are realized in another one (response). Once the poles have been selected it is possible to estimate the vibration shape **[12, 13]**.

3. Description of the subject of study

The study is applied to the passenger vehicles belonging to a railway vehicle fleet of public transportation of Medellín city (Colombia), similar to a suburban train system (*Fig. 1a*) **[13]**.

3.1. Background of the railway vehicle

The vehicle is a three-units cars, each of which have two (2) bogies. Each bogie is supported over two (2) axle-wheel set, and each car has two (2) suspension stages: primary and secondary.

The primary suspension stage is composed of external and internal helicoidal springs, parallel to each helicoidal spring there is an hydraulic damper in vertical direction, and two (2) guide leafs are disposed on the end of the axle (*Fig. 1b*) providing the guide to the axlebox **[14, 15]**.

The airsprings are part of the secondary suspension stage (*Fig. 1b*) and aim to fix the height of the carbox. Each airspring is mounted on an auxiliary spring that works in case of failure of the first one **[15]**. The vertical suspension is done through the hydraulic damper associated to each airspring.

This work is focused on the study of the dynamic vehicle response [11], which is influenced by the technical state of the four (4) identical vertical dampers that each car has δ_j with j = 1,...,4. This type of component is characterized by a set of physical laboratory tests. The method of the test requires the application of an excitation of cyclical displacement (sinusoidal) in ranges of frequency and determined peak-to-peak amplitudes **[4, 15]**. The obtained relationship from the characterization of the element is denominated *nominal damping function*, $\in_{10}$. From $\in_{10}$ a set of nine (9) hypothetical properties are established, $\in_i$ with i = 1,...,9; which present a behavior similar to $\in_i$, but affected by a coefficient that reduces the suspension properties of the element δ_j, $\epsilon_i = \frac{i}{10}\epsilon_{10}$. In this way there are different technical states of the component, generating a progressive degradation of the damping function (*Fig. 1c*).

3.2. Evaluation criteria

The instability of a vehicle must be avoided in the range of travel speed V, otherwise there will be an accentuated effect in the oscillation frequency ω that generates a dynamic negative behavior or *Hunting* that can even cause derailment. The problems of non-linear stability of railway vehicles have been subject of great amount of study and investigation **[16–18]**.

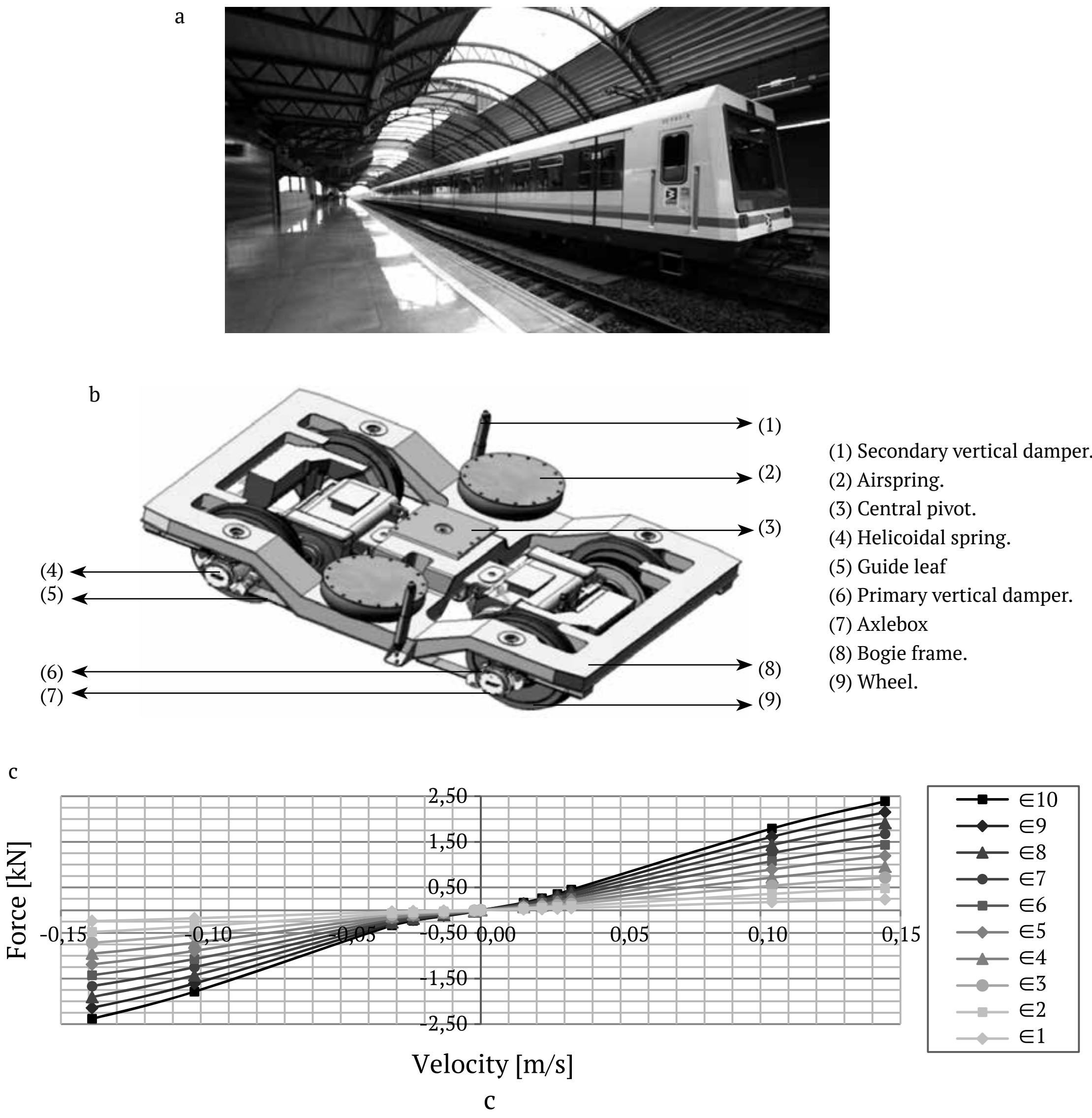


Fig. 1. **Railway vehicle: a. three-unit cars trainset; b. suspension elements of the tow (2) stage vehicle; c. damping function of the secondary damper**

The damping must be adjusted to control the behavior of the vehicle operating at a speed inferior to the critical speed V_{cr}, $V_{cr} > V$ **[19, 20]**. The stability must be determined through tests and simulations. Literature has established different limit values, relative to ζ **[4, 20, 21]**: (i) danger limit, $\zeta_{\lim 1} = 0\,\%$ a vehicle with a value $\zeta \le 0\,\%$ implies that the system excites itself, generating instability **[20, 22]**; and (ii) alarm limit $\zeta_{\lim 2} = 5\,\%$, a vehicle is considered stable if it has a value ζ of 5 %, on the contrary it is denominated unstable (unsatisfactory) **[23]**.

The unfavorable conditions that go in detriment of passengers comfort is defined according to the ζ and ω criteria **[24, 25]**: (i) unsatisfactory limit, $\zeta_{\lim 1} = 5\,\%$, value $\zeta >$ 5 % determines a positive perception of the passengers; (ii) deficient frequency range, ω_r = [8,10] Hz, the human body is sensible to vertical accelerations in this range; and (iii) limit of balance oscillation, $\omega_2 = 0{,}5$ Hz, the vehicle must have a value ω superior to 0,5 Hz, on the contrary there is the risk to produce a movement that causes nausea to passengers.

3.3. Construction of a failure hypothesis

Three (3) hypothesis ζ_i are established in relation with the state of the suspension elements δ_j, defined parametrically through the progressive degradation of the *damping fuction*:

- hypothesis ζ_1: consists on the reduction of the *damping function*, changing the technical state $\in_i$ of the all vertical hydraulic secondary dampers δ_j, with $j = 1,\ldots,4$;
- hypothesis ζ_2: consists on the reduction of the *damping function*, changing the technical state $\in_i$ for the leading dampers, this is the ones located to at a near to the driver (δ_1, δ_2). The others dampers (δ_3, δ_4) conserve the nominal property $\in_{10}$; and
- hypothesis ζ_3: consists on the reduction of the *damping function*, changing the technical state $\in_i$ for the training dampers, this is the ones located to at the far end opposite to the driver (δ_3, δ_4). The others dampers (δ_1, δ_2) conserve the nominal property $\in_{10}$.

4. Dynamic characterization of the vehicle applying the EMA method

The EMA technique has been used to determine the vehicle behavior in a given operating condition. The excitation of the vehicle is done trough a manual excitation on the carbox, on the location and direction necessary to induce a specific shape mode; to generate that such excitation, it is necessary to remove the damping elements from the vehicle suspension, condition denominated null damping condition, $\in_0$. The acquisition of the data is done with a minimum sampling frequency of 50Hz **[13]**. The set of signals is transformed to the domain of the frequency with the FFT algorithm to obtain the characteristic frequencies of the carbox and therefore the corresponding modal shape **[4, 26]**. *Table 1* presents the results obtained with the EMA technique.

5. Development of numeric models

The railway vehicle model can be developed and running on a typical track and instrumented in a virtual environment, which allows investigating the effects of a wide range of possible variations of the vehicle parameters **[4, 11]**. The obtained results of a model can provide certain predictions of the dynamic behavior of the vehicle and the interaction with the track **[27]**.

The focus of a virtual model is to numerically integrate the ordinary differential equation that constitutes the model, using one or various integrating algorithms. This focus is usually known as simulation or numeric experimentation, which is equivalent to physics experimentation where the system is subject to given conditions and a response is recorded.

The numeric model is used to simulate the characteristics of the system and is considered a real virtual prototype. Virtual techniques allow the model to generate information about the dynamic behavior and the interaction between components, which are comparable only to physical prototypes. Simulations with numeric models are a valid source of necessary data to prove the formulated methodologies. The virtual techniques allow the model to generate a great quantity of information regarding not only the response and the dynamic behavior, but also about the interaction of the components, capacity details, and even esthetic qualities, which are only comparable to physical prototypes **[3, 4, 12, 27]**.

The numeric model developed counts with 120 DoF **[15]** and comes from the union of the two (2) simplified models (*Fig. 2a*) and represents a complete motor car (*Fig. 2b*). It is based on theory of multi-body systems, using the analysis software VAMPIRE®. The values of the particular parameters of the model are exposed on Appendix 2.

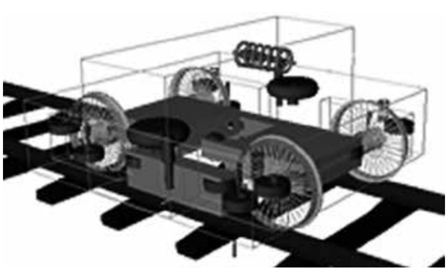
6. Development of OMA-LSCE method in a virtual environment

It is necessary to point out the guidelines in order to be able to realize the test properly. The coherent design of the experiment allows a correct analysis of the data; this must be designed to evaluate the secondary suspension under controlled and normal operating conditions, without affecting the operation and security of the

Table 1. **Modal parameters identified with the EMA technique, $\in_0$ condition.**

Modal shape, φ_r	Lower roll, φ_1	Bounce, φ_2	Yaw, φ_3	Pitch, φ_4
Natural frequency, φ [Hz]	0,80	1,47	1,50	2,10

a



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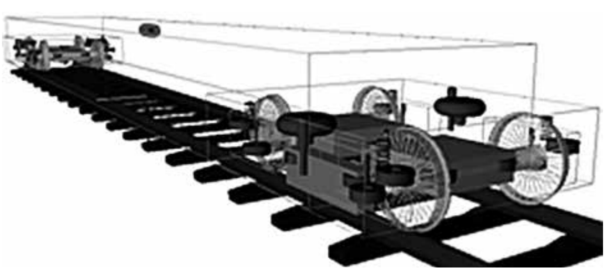


Fig. 2. **Graphic representation of the numeric models: a. simple model; b. complete model**

system. The test is defined following these conditions:

• the load condition of the vehicle (AW0), meaning the car is empty;

• the track section is straight line and track irregularities are considered;

• the vehicle speed, V= 80 km/h; and

• a variation of the technical state of secondary suspension dampers is assumed $\in_i$ (*Fig. 1c*).

The measurement points are defined according to the guidelines of the railway standards **[22]**. Due to the fact that the interest of the work is focused on the study of the secondary vertical suspension, the signals to be registered must in the main bodies of the system, in this way three (3) signals in vertical direction are chosen. The signals are located:

• $\ddot{Z}^*_{ql}$, on the carbox floor, near to the driver;

• $\ddot{Z}^+_{ql}$, in the bogie frame near to the attack axle; and

• $\ddot{Z}^o_{ql}$, in the axlebox of the attack axle.

The set of signals $\ddot{Z}^*_{ql}$, $\ddot{Z}^+_{ql}$, and $\ddot{Z}^o_{ql}$ is processed by the OMA-LSCE method, based on the measurement of response of the system to the own excitations of the operation and from which the global modal parameters are estimated: δ, ξ and φ **[12]**. The modal parameters are graphically represented by the stabilization diagrams. Appendix 3 exposes as an example a set of stabilization diagrams obtained from different operating conditions of the vehicle. In the diagrams it is possible to identify:

• two (2) local maximums in the range [1, 2.5] Hz, which correspond to the two (2) modal shapes (φ_2, and φ_4), and they are registered in the carbox by the sensor $\ddot{Z}^*_{ql}$;

• three (3) local maximums in the range [4, 10] Hz, which correspond to the three (3) modal shapes recorded in the bogie by the sensor $\ddot{Z}^+_{ql}$; and

• three (3) local maximums in the range [15, 25] Hz, which correspond to the vertical irregularities of the track, and they are obtained from the signal recorded by the sensor $\ddot{Z}^o_{ql}$.

The work focuses on the analysis of the vehicle, more specifically in the carbox for which it has been possible to identify two (2) vertical modal shapes (φ_2, φ_4) under the load condition AW0. From the set of values of the modal parameters identified for the different hypothesis ζ_i it is possible to observe the existing relationship between the technical state of the damping set and the modal shapes φ_2 y φ_4, obtaining regressive lineal models with a correlation coefficient value of $\sqrt{R^2} > 0{,}98$ (*Fig. 3*).

The regressive models are considered valid given that the values $\sqrt{R^2}$ represent an association measurement of the statistical model with the obtained data [26], which have an acceptable level for the reach of this work.

7. Validation of the numeric models

The regressive models belonging to ζ_1 and obtained by OMA-LSCE in a virtual environment are extrapolated to the value of the technical state of the damper with a null damping function, $\in_0$, for the modal shapes φ_i, meaning:

$$\text{for } \varphi_2, \in(\Omega) = 9{,}577\Omega - 13{,}950 \;\therefore\; \text{if } \in = \in_0 \rightarrow \Omega = 1{,}46 \text{ Hz},$$

$$\text{for } \varphi_4, \in(\Omega) = 8{,}761\Omega - 18{,}326 \;\therefore\; \text{if } \in = \in_0 \rightarrow \Omega = 2{,}09 \text{ Hz}.$$

In this way the dynamic behavior of the vehicle without the secondary damper is obtained, emulating the total extraction of the damping elements of the suspension δ_j with j = 1,…,4; which is the same testing condition realized to the vehicle with the EMA technique.

The values registered by the two (2) types of analysis, EMA and OMA, are compared (*Table 2*) which allows to observe that

Table 2. **Parameters identified with the EMA and OMA techniques, condition $\in_0$.**

Modal analysis type	Ω, Hz	
	φ_2	φ_4
EMA	1,47	2,10
OMA	1,46	2,09
Error, ε [%]	0,91	0,04

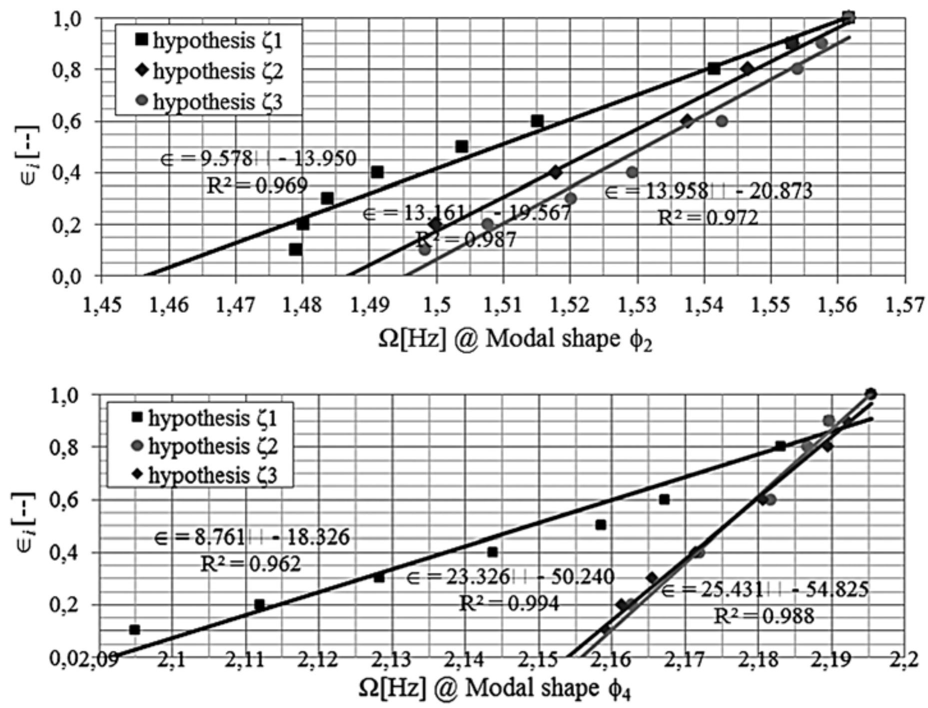


Fig. 3. **Development of regressive models ζ_i obtained with the modal parameter Ω**

there is an estimation error of the modal parameter $\varepsilon < 1\%$. Therefore, it is possible to consider valid the regressive models given that the ε values represent an acceptable deviation level for the reach of this work.

8. Identification of the technical state of the suspension using the OMA-LSCE method

OMA-LSCE method is applied to the signals acquired during a test done to the three-car unit cars under typical operation on a commercial track [4]. The measurement points are defined according to the guidelines of the railway standards. Given that the interest of this work focuses on the study of the secondary vertical suspension, the signals to be registered must be in the main bodies of the system, therefore three (3) signals in vertical direction are chosen. The signals are located: (i) $\ddot{Z}^{*}_{ql}$, *Fig. 4a*; (ii) $\ddot{Z}^{+}_{ql}$, *Fig. 4b*; and (iii) $\ddot{Z}^{o}_{ql}$, *Fig. 4c*.

There are three (3) straight commercial track sections denominated [15]: *Recta58–63, Recta78–79* and *Recta85–87*. The signal $\ddot{Z}^{o}_{ql}$ have a wide and soft band, meaning that the PS is constant and it has not poles or zeros in the frequency range of interest. Additionally, it is especially distributed in a uniform way, as [29]. The results of the section *Recta58–63* (*Fig. 5*) are presented as an example. The respective discrete PSD function is calculated $S(\omega)$ (*Fig. 5a*), observing that the function $S(\omega)$ of the $\ddot{Z}^{*}_{ql}$ signal only has one local maximum in the range of study [0, 25] Hz, which means that

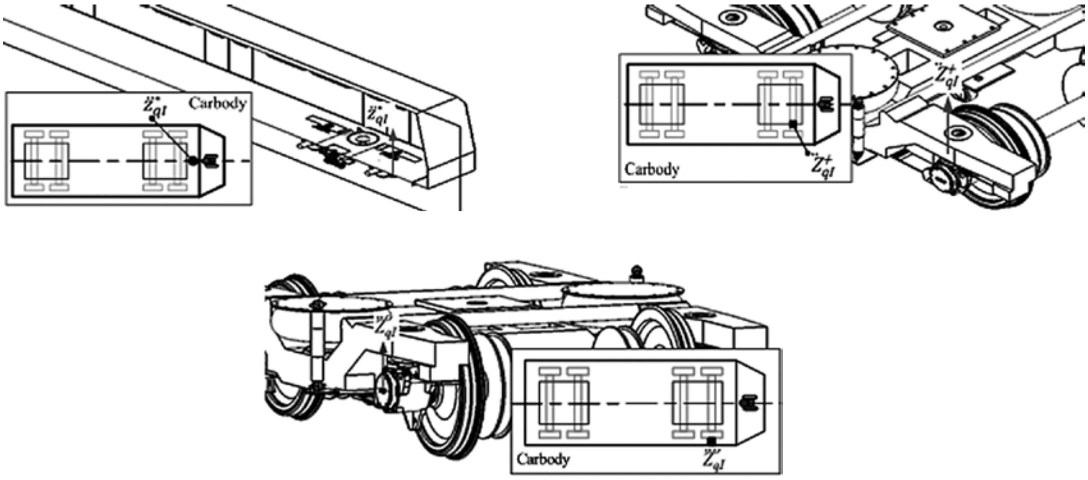
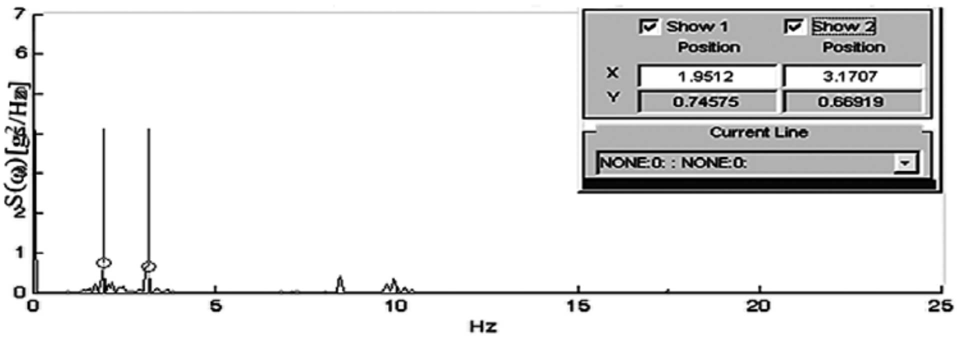
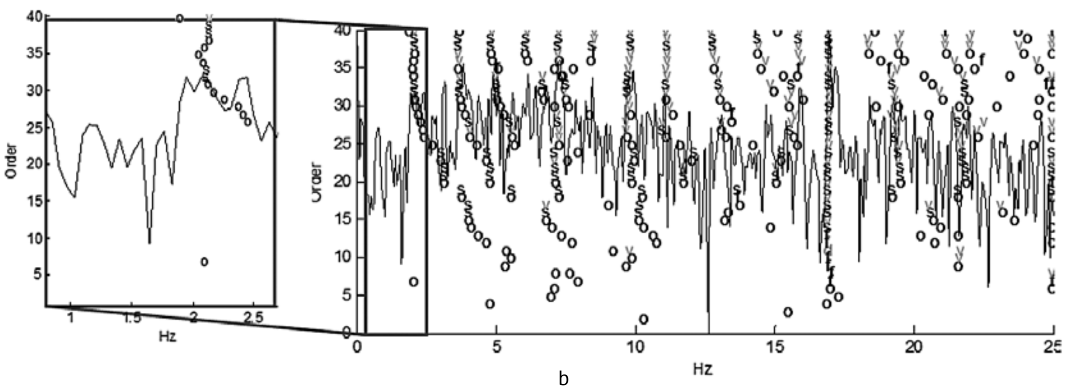


Fig. 4. **Location of the sensors in: a. carbox $\ddot{Z}^{*}_{ql}$; b. bogie frame $\ddot{Z}^{+}_{ql}$; and c. axlebox $\ddot{Z}^{o}_{ql}$.**



a



b

Fig. 5. **Result of the modal identification in the section track *Recta*58–63: a. PSD of $\ddot{Z}^{*}_{ql}$; and b. Diagram of stabilization of $\ddot{Z}^{*}_{ql}$**

Table 3. **Modal parameters identified by OMA**

Track section	Order	Ω, Hz	ζ, %	φ_i
Recta 58–63	32	2,114	12,38	φ_4
Recta 78–79	38	1,544	19,38	φ_2
Recta 85–87	30	1,944	15,97	φ_4

the perturbation-response relation of the system is able to excite properly only one of the two vertical modal shapes (φ_2, or φ_4) depending on the particular irregularities of the track and the corrugation of the rail in the instant of the registration of the signals. Based on function $S(\omega)$ the stabilization diagram is obtained **[10]** (*Fig. 5b*). As $S(\omega)$ only registers one modal shape φ_i, the LSCE method identifies only the parameters associated to that shape (φ_2, o φ_4).

The results of the application of OMA-LSCE method are presented in *Table 3*, obtaining the following modal parameters: (i) identification of φ_2, $\Omega \cong 1{,}53$ Hz and $\zeta \cong 19{,}3\%$; and (ii) identification of φ_4, $\Omega \cong 2{,}18$ Hz; and $\zeta \cong 14{,}1\%$.

Based on the railway criteria for on motion security and passengers comfort, it is possible to perform an evaluation of the vehicle. The following relations are found: $\Omega \notin \omega_r, \zeta >> \zeta_{\lim 1}$, y $\zeta > \zeta_{\lim 2}$; therefore, it's valid to affirm that the vehicle satisfies the requirements of the international standards for the two (2) criteria studied in the present work regarding the oscillation shapes φ_2, y φ_4.

Once the set of signal of one car have been obtained $\ddot{Z}^*_{ql}$, $\ddot{Z}^+_{ql}$, y $\ddot{Z}^o_{ql}$, and the data processed through OMA-LSCE, the following identification of modal parameters is obtained: natural frequency: $\Omega = 1{,}53$ Hz, for φ_2 and $\Omega = 2{,}18$ Hz, for φ_4.

Starting from the identified modal parameters and based on the series of regressive models obtained for the vehicle, the probable state of the damper is located $\in_i$, for each one of the hypothesis ζ_i. The regressive models in the identification of the modal shape φ_2 are:

$$\text{for } \zeta_1, \in(\Omega) = 9{,}577\Omega - 13{,}950 \therefore \in(1.53) = 0{,}70,$$

$$\text{for } \zeta_2, \in(\Omega) = 13{,}161\Omega - 19{,}567 \therefore \in(1.53) = 0{,}57,$$

$$\text{for } \zeta_3, \in(\Omega) = 13{,}958\Omega - 20{,}873 \therefore \in(1.53) = 0{,}48;$$

In the modal shape φ_4 are:

$$\text{for } \zeta_1, \in(\Omega) = 8.761\Omega - 18.326 \therefore \in(2.18) = 0{,}77,$$

$$\text{for } \zeta_2, \in(\Omega) = 23.326\Omega - 50.240 \therefore \in(2.18) = 0{,}61,$$

$$\text{for } \zeta_3, \in(\Omega) = 25.431\Omega - 54.825 \therefore \in(2.18) = 0{,}61.$$

The $\in$ values from the hypothesis ζ_i are tabulated (*Table 4*). The mean of the technical state is calculated, $\bar{X}_i$ as well as their corresponding standard deviation σ_i, the later will be the criteria that defines the valid hypothesis ζ_t. The ζ_i that presents the least standard deviation is the one that adapts to the obtained dynamic characteristics of the system and therefore the value $\bar{X}_i$ of such hypothesis must be the technical state of the damper $\in$, meaning $\zeta_t = \bar{X}_i(\in)\nabla\min(\sigma_i)$.

The technical state of the damper is $\in = \bar{X}(\in) \mp 2\sigma$ with 95,45 % of confidence factor. For the given case study, the value of the technical state is $\in = 0{,}71\in_{10}\pm 0{,}1$ in the four secondary vertical dampers (hypothesis ζ_1) with a certainty of 95,45 %. This means that the dampers are degraded 29 %.

9. Conclusions

The OMA-LSCE method is a suitable tool to establish the properties of the system components necessary for the evaluation of the vehicle according to the UIC, ISO and BS standards. Additionally, the formulated methodology is valid for establishing the technical state of the dampers $\in_i$, through dynamic signals obtained in the commercial service of the system.

A methodology has been proposed and applied for the evaluation of the technical state of the dampers $\in$ and the identification of hypothesis of suspension elements deterioration ζ_i through measurements of variables under operating conditions of the vehicle.

The vertical dampers of the secondary suspension stage present a direct influence on the dynamic behavior of the modal shapes φ_2 and φ_4, therefore the different technical states of the damper $\in_i$ can be tested and estimated through the dynamic registration in the carbox. This means that from the sensors installed in the carbox that register appropriately the natural frequency Ω it is possible to determine the variation of the damping function of dampers, in this way it is possible to infer their degradation or failure.

Table 4. **Evaluation of the technical state $\in$, in order to ζ_i**

Modal shape, φ_i	Modal parameter		Damper technical state, $\in$ [–]		
	Description	Value	ζ_1	ζ_2	ζ_3
φ_2	Ω, Hz	1.53	0,70	0,57	0,48
	ξ,%	2.90	0,70	0,58	0,42
φ_4	Ω, Hz	2.18	0,77	0,61	0,61
	ξ,%	2.20	0,67	0,75	0,72
Mean value, $\bar{X}_i$			0,71	0,63	0,56
Standard deviation, σ_i			0,04	0,08	0,14

The dynamic parameters, δ and ξ, have a high dependence degree over the secondary suspension dampers δ_j.

Three (3) suitable sections track have been identified for the computation of the OMA-LSCE method through experimental registrations of the system in commercial operation, under an experimental model of three (3) measurement points, $\ddot{Z}^{*}_{ql}$, $\ddot{Z}^{+}_{ql}$, y $\ddot{Z}^{o}_{ql}$ obtained from a set of sensors. The signals from the sensors can be used to identify the dynamic parameters, Ω and ξ, in a vehicle with load condition AW0.

The experimental recording under load condition AW0 can be used to determine the technical state of the system components $\in_i$ with the OMA-LSCE method. The OMA-LSCE method it is not efficient for the identification of the technical state of the secondary dampers of a vehicle with load condition AW2 given that the perturbation-response relation is not enough to obtain a clear identification of the modal shapes φ_i.

The secondary suspension dampers have a high degree of dependence over the on motion security and passengers comfort criteria. International standards for railway vehicles define the range for deficient frequency ω = [8, 10] Hz. The human body is sensible to vertical accelerations; frequencies $\omega \approx 10$ Hz cause excessive oscillations on φ_2, generating significant deficiency of comfort. Comparing with the values obtained for the analyzed vibration modes φ_r and under the different technical states of the damper $\in_i$, the natural frequency is $\Omega < 2$ Hz. Therefore, the degradation of the damping function of the suspension elements does not incur *per se* in violation to the railway standards.

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Appendix 1. Notation

Abbreviations, acronyms, coefficients and constants

AR	Auto-regression model.
AW0	*Anhängerladung Wägestück 0* (load weight 0; empty car).
DC	Direct Current.
DoF	Degrees of Freedom.
EMA	Experimental Modal Analysis.
FFT	Fast Fourier Transform.
FRF	Frequency Response Function.
FT	Fourier Transform.
IRF	Impulse Response Function.
LSCE	Least-Squares Complex Exponential.
MAN	*Maschinenfabrik Augsburg-Nüernberg.*
MDoF	Multiple Degrees of Freedom.
MIMO	Multiple Input Multiple Output.
ODS	Operating Deflection Shapes.
OMA	Operational Modal Analysis.
FT	Fourier Transform.
RD	Random Decrement process.
${}_rA_{ij}$	rth modal constant.
h_k	kth IRF.

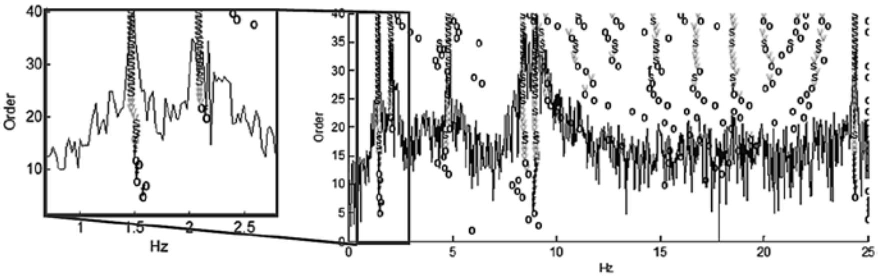
kΔ	Time interval.
R^2	Coefficient of determination.
$\sqrt{R^2}$	Coefficient of correlation.
s_r	rth complex quantity root.
$\bar{X}_i$ $i = 1, 2, 3$	ith mean
V	Vehicle speed.
z_r	Conjugacy of the roots s_r.
$\ddot{Z}^{*}_{qP}, \ddot{Z}^{+}_{qI}, \ddot{Z}^{o}_{qI}$	Vertical acceleration at carbody, leader bogie and attack axlebox, respectively.
Ω	Natural frequency.
$\alpha\ (\omega)$	Receptance matrix
$\beta_k k = 0,1, \ldots, 2$	kth real coefficient.
$\delta_j j = 1, \ldots, 4$	jth secondary vertical damper.
ε	Error value.
$\in_i i = 1,\ldots,10$	ith damper technical state.
φ_r	rth mode shape.
$\sigma_i i = 1,2,3$	ith standard deviation
Ω	Oscillation frequency.
ξ	Damping rate.
$\zeta_i i = 1,2,3$	ith hypothesis.

Appendix 2. Vehicle parameters

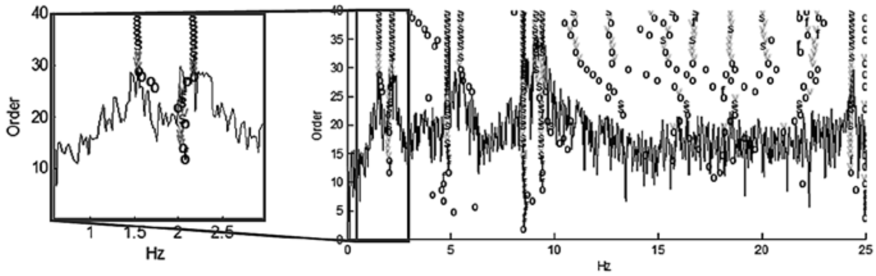
Element	Quantity	Value	Unit
Mass			
Carbody	1	24486.36	kg
Bogie frame	2	6102.44	kg
Motor	4	5551	kg
Electromagnetic brake	4	704	kg
Traction link	2	–	kg
Axle-wheel set	4	7083,52	kg
Stiffness			
Lineal	1	$k_A = 1{,}00$	kN/mm
Non-lineal	20	–	kN/mm
Shear	8	$k_x = 2{,}16$	kN/mm
		$k_y = 2{,}16$	kN/mm
		$k_z = 12{,}16$	kN/mm
Axi-directional	2	0,08	kNmm/s
Air stiffness	4	$k_z = 4{,}52$	kN/mm
		$k_y = 1{,}04$	kN/mm
Non-lineal damper	13	–	kNmm/s
Busing	36	$k_x = 67{,}32$	kN/mm
		$k_y = 36E-5$	kN/mm
		$k_z = 72E-6$	kN/mm
		$k_\theta = 36E-5$	MNm/rad
		$k_\varphi = 2E-6$	MNm/rad
		$k_\psi = 1E-5$	MNm/rad
DOF	120	–	–
Conicity non-lineal		–	–

Appendix 3. Stabilization diagrams

a. Hyphotesis ζ_1 with $\in_1$; b. Hyphotesis ζ_1 with $\in_9$; c Hyphotesis ζ_2 with $\in_1$;
d. Hyphotesis ζ_2 with $\in_9$; e. Hyphotesis ζ_3 with $\in_1$; f. Hyphotesis ζ_3 with $\in_9$.



a



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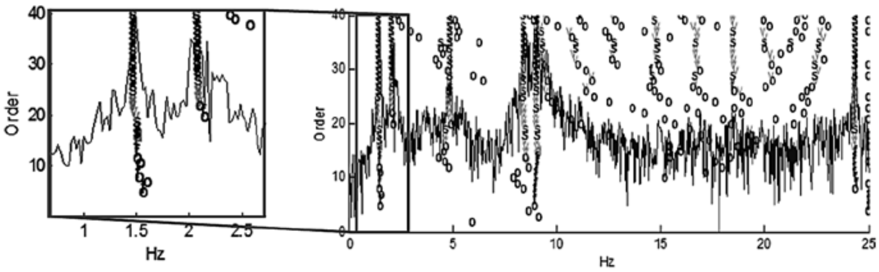

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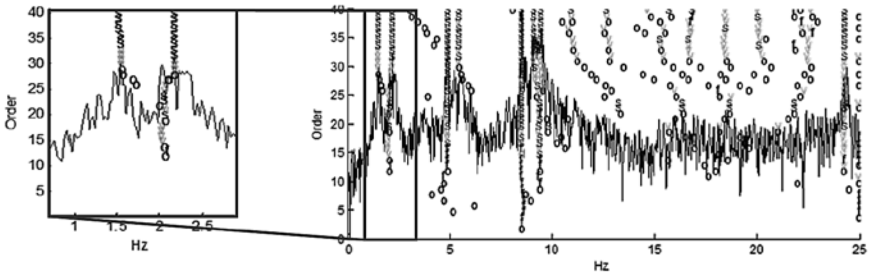

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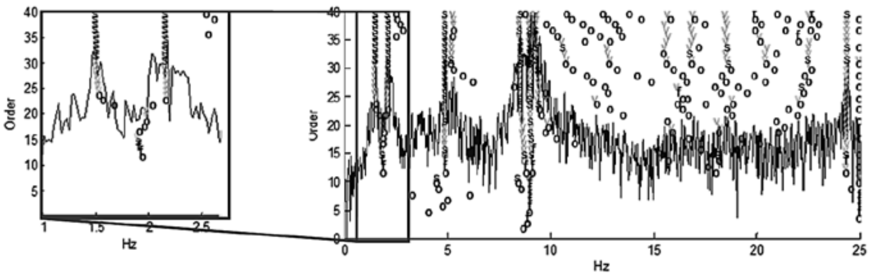

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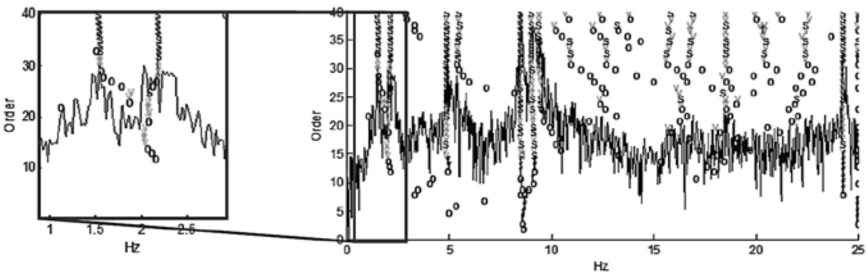

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